Current fluctuations in boundary driven diffusive systems

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We consider diffusive systems whose boundaries are connected to particle reservoirs at different densities. The object of interest is the net number of particles that have passed through the system in the steady state, during a time interval $[0,t]$, denoted by Q_t. Because the dynamics is stochastic, the particle flow is a fluctuating quantity described by a probability distribution $P(Q_t)$. This distribution is studied in the thermodynamic limit, i.e., for large system sizes and for large times t. There has recently been a strong interest in the study of current fluctuations in the field of nonequilibrium statistical mechanics. A proper understanding of these fluctuations could allow one to define nonequilibrium thermodynamic potentials, in a conceptually similar way as for equilibrium potentials [1].

If one knows the transport diffusion $D(\rho)$ and the mobility $\sigma(\rho)$, one can calculate the full current distribution in a one-dimensional system. The transport diffusion $D(\rho)$ is defined as

$$\frac{\langle Q_t \rangle}{t} = -D(\rho)\frac{\partial \rho}{\partial x}, \tag{1}$$

with $\langle \cdot \rangle$ the average over $P(Q_t)$, and the concentration gradient small enough so that linear response is valid. The mobility quantifies equilibrium fluctuations of the current

$$\frac{\langle Q_t^2 \rangle}{t} = \frac{1}{L}\sigma(\rho), \quad \rho_A = \rho_B = \rho, \tag{2}$$

with ρ_A and ρ_B the densities of the two reservoirs, and L the length of the system. All the moments of the current distribution can be calculated from the integral [2]

$$I_m = \int_{\rho_B}^{\rho_A} D(\rho)\sigma(\rho)^{m-1}\, d\rho. \tag{3}$$

For example, the variance of the particle flow is equal to

$$\frac{\langle Q_t^2 \rangle - \langle Q_t \rangle^2}{t} = \frac{1}{L}\frac{I_2}{I_1}. \tag{4}$$

Akkermans and co-workers performed a theoretical calculation that showed that the above result can also be used to predict current fluctuations in higher-dimensional systems [3]. This result is valid for arbitrary shapes of the system and of the contacts between the reservoirs and the system, as long as both are of macroscopic size.

We present numerical results, obtained from kinetic Monte Carlo (kMC) simulations, for the current fluctuations in a stochastic lattice gas model that was introduced in [4]. Our results [5] are in full agreement with the analytical predictions from [3]. Converge of the Fano factor, $F = (\langle Q_t^2 \rangle - \langle Q_t \rangle^2) / \langle Q_t \rangle$, to the analytical prediction for a two-dimensional lattice gas with a non-trivial coupling to the reservoirs is shown in Figure 1.

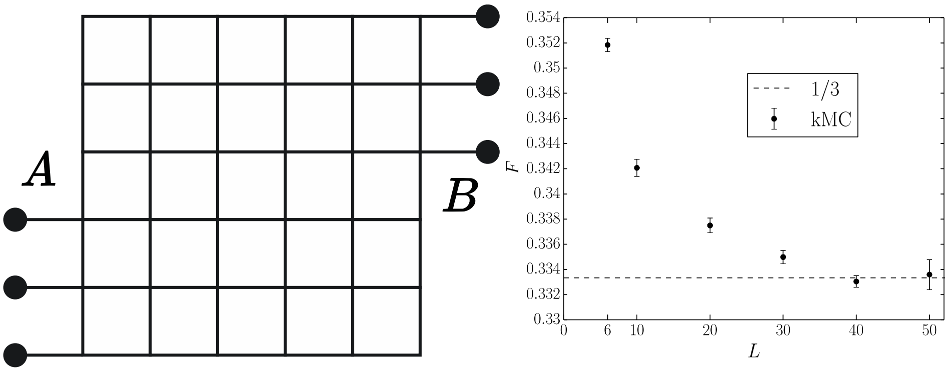

Figure 1: (left) A square lattice with sides of length L connected on its left and right to particle reservoirs A and B (black dots). Each site can contain maximally one particle. The reservoirs have densities $\rho_A = 1$ and $\rho_B = 0$. The lower left half and the upper right half of the system is connected to the reservoirs. (right) The Fano factor as a function of the length L, with one-sigma error bars. Convergence to the analytical prediction $F = 1/3$ is found for $L \geq 40$.

For a general diffusive system, the transport diffusion depends on the dimension, and the current statistics changes for different dimensions. However, the one-dimensional theory can still be used to predict the full current distribution. This is shown in Figure 2 for a stochastic lattice gas where each site can contain maximally two particles.

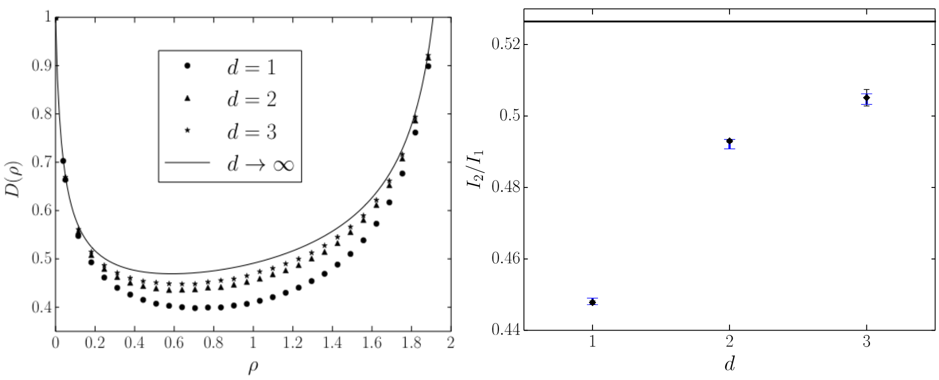

Figure 2: (left) Numerically obtained transport diffusion in one, two, and three dimensions. The limiting case of infinite dimensions is shown as a line. Error bars are smaller than the symbol sizes. (right) The variance of the current $L(\langle Q_t^2 \rangle - \langle Q_t \rangle^2)/t$ in different dimensions, obtained from kMC simulations (black diamonds) and the prediction from equations (3) and (4) (blue error bars). The limiting case of infinite dimensions is shown as a line. All symbols have one-sigma error bars.

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