On the geometrical description of effective diffusion in confined environments: two-dimensional case

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In this contribution we provide a formal differential geometrical description of diffusion of Brownian particles in confined two-dimensional systems. We present a method to map the boundaries of a general two-dimensional channel in Cartesian coordinates, into a straight channel in a non-Cartesian geometry [1]. To this end we propose a coordinate transformation where the new boundaries naturally suggest a reduction to one dimension, so the diffusion equation turns to be a generalized Fick-Jacobs-like equation. The expression for the effective diffusion coefficient obtained by our method depends on the position and the derivatives of the channel's width and midline. To obtain corrections on the effective diffusion coefficient we performed an expansion of the concentration in terms of the centerline up to second order.

We present some examples for the effective diffusion coefficients of periodic channels through the Lifson-Jackson formula. In those cases, the effective diffusion coefficient corresponding to our second-order approximation considerably improves the zeroth-order coefficient, which reduces to all previously studied cases in the literature [2].

The three-dimensional case will be presented in another contribution. This analysis results in a general method to describe the confined diffusion.

References

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